香港國際數學競賽初賽 **2019**（香港賽區）

HONG KONG INTERNATIONAL MATHEMATICAL OLYMPIAD HEAT ROUND 2019 (HONG KONG REGION)

中學三年級 Secondary 3

時限：90 分鐘

Time allowed: 90 minutes

模擬試題

Mock Paper

考生須知：

Instructions to Contestants:

1. 本卷包括 試題 乙份，試題紙不可取走。

   Each contestant should have ONE Question-Answer Book which CANNOT be taken away.

2. 本卷共 5 個範疇，每範疇有 5 題，共 25 題，每題 4 分，總分 100 分，答錯不扣分。

   There are 5 exam areas and 5 questions in each exam area. There are a total of 25 questions in this Question-Answer Book. Each carries 4 marks. Total score is 100 marks. No points are deducted for incorrect answers.

3. 請將答案寫在 答題紙 上。

   All answers should be written on ANSWER SHEET.

4. 比賽期間，不得使用計算工具。

   NO calculators can be used during the contest.

5. 本卷中所有圖形不一定依比例繪成。

   All figures in the paper are not necessarily drawn to scale.

6. 比賽完畢時，本試題會被收回。

   This Question-Answer Book will be collected at the end of the contest.

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

填空題（第 1 至 25 題）（每題 4 分，答錯及空題不扣分）

Open-Ended Questions (1st ~25th) (4 points for correct answer, no penalty point for wrong answer)

Logical Thinking
邏輯思維

1. Given A and B are three non-zero digits and the 3-digit numbers formed by these three digits have the following properties:
   1. $\overline{BBA}$ is divisible by 17;
   2. $\overline{AAB}$ is divisible by 7;
   3. $\overline{ABA}$ has an odd number of factors.
   Find the 3-digit number $\overline{BAB}$.

   已知 A 和 B 為三個非零數位且利用這三個數位組成的三位數有以下性質：
   1. $\overline{BBA}$ 可以被 17 整除；
   2. $\overline{AAB}$ 可以被 7 整除；
   3. $\overline{ABA}$ 的因數數目為單數。

   求三位數 $\overline{BAB}$。

2. Given that the mean, median, range and the only mode of 120 integers are also 70. If A is the largest integer among those 120 integers, find the maximum value of A.

   已知 120 個整數的平均數、中位數、分佈域及唯一眾數皆為 70。若 A 為該 120 個整數中的最大整數，求 A 的最大值。

3. Andy goes west for 13km, then goes north for 26km, goes east for 61km and goes south for 46km. How far is he now from the original position?

   安迪向西走了 13 公里，向北走了 26 公里，向東走了 61 公里，向南走了 46 公里，問他和原來位置相距多遠？

4. There are 28 problems in a mathematics competition. The scores of each problem are allocated in the following ways: 2 marks will be given for a correct answer, 1 mark will be deducted from a wrong answer and 0 marks will be given for a blank answer. Find the minimum number of candidate(s) to ensure that 3 candidates will have the same scores in the competition.

   某數學比賽共有 28 條題目。以下述方式為每個題目評分：答對得 2 分、答錯得零分、不作答倒扣 2 分。求最小參賽者的數目才可保證比賽中有三人同分。

5. There are 10 pieces of white chopsticks, 8 pieces of yellow chopsticks and 9 pieces of brown chopsticks

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mixed together. Close your eyes. If you want to get 2 pairs of chopsticks that are not yellow, at least how many piece(s) of chopstick(s) is / are needed to be taken?

現有白色、黃色、棕色的筷子分別有 10 根、8 根和 9 根，把它們混在一起。你要在閉上眼睛的情況下，從這些筷子中取出 2 雙不是黃色的筷子，最少要取多少根？

Algebra
代數

6. Let α and β ($\alpha, \beta \neq 0$) be the roots of the equation $x^2 - 2018x + 1 = 0$. If $\dfrac{1}{\alpha^2 \beta}$ and $\dfrac{1}{\alpha \beta^2}$ are the roots of the equation $x^2 - Sx + 1 = 0$, find the value of S.

已知 $\alpha, \beta \neq 0$，設方程 $x^2 - 2018x + 1 = 0$ 的根為 α 和 β。若 $\dfrac{1}{\alpha^2 \beta}$ 和 $\dfrac{1}{\alpha \beta^2}$ 是方程 $x^2 - Sx + 1 = 0$ 的根，求 S 的值。

7. Factorize $xy^2 + yz^2 + zx^2 - x^2 y - y^2 z - z^2 x$.

因式分解 $xy^2 + yz^2 + zx^2 - x^2 y - y^2 z - z^2 x$。

8. Simplify $\sqrt{30 + 12\sqrt{6}}$.

化簡 $\sqrt{30 + 12\sqrt{6}}$。

9. One integral roots of equation $\left| x^2 + 6x + a \right| - 5x = 3$ is 1. Given that $a > 0$, find the other integral root.

方程 $\left| x^2 + 6x + a \right| - 5x = 3$ 的其中一個整數解是 1，已知 $a > 0$，求另一個整數解。

10. Find the greatest value of $\dfrac{901}{x^2 + 40x + 401}$.

求 $\dfrac{901}{x^2 + 40x + 401}$ 的最大值。

Number Theory
數論

11. Given that $\overline{81A02B}$ is a 6-digit number which is divisible by 36 and $B > A$, find the value of $A + B$.

已知 $\overline{81A02B}$ 是一個六位數，且可被 36 整除及 $B > A$，求 $A + B$ 的值。

12. Find the remainder for 122^{122} divided by 9.

求 122^{122} 除以 9 的餘數。

13. How many positive integer(s) x that is less than 8012 is / are there so that $\sqrt{x-2018}$ is an integer?

    有多少個小於 8012 的正整數 x 使得 $\sqrt{x-2018}$ 為整數？

14. Now is June. Which month is it after 15^{15} months?

    現在是六月，15^{15} 個月後是幾月？

15. It is given that x and y are real numbers such that $x+y=6$ and $x^2+y^2=10$, find the value of x^3+y^3.

    已知 x 及 y 皆為實數使 $x+y=6$ 及 $x^2+y^2=10$。求 x^3+y^3 的值。

Geometry
幾何

16. Find the area enclosed by the *x*-axis, *y*-axis and the straight line $2y = 3x + 12$.

    求由 *x*-軸、*y*-軸及直線 $2y = 3x + 12$ 圍出的面積。

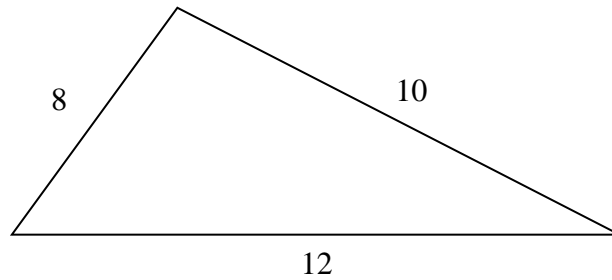17. Find the maximum value of $\sin x + \cos x$.

    求 $\sin x + \cos x$ 的最大值。

18. Given that an interior angle of an *n*-sided regular polygon is 17 times an exterior angle. Find the value of *n*.

    已知一正 *n* 邊形其中一個內角是一個外角的 17 倍。求 *n* 的值。

19. A triangle in the figure below has sides with lengths 8, 10 and 12. Find the value of the radius of inscribed circle of that triangle.

    下圖中的三角形的邊長分別是 8、10 和 12。求內接於該三角形的圓形的半徑。

Question 19
第 19 題

20. Find the shortest distance from the straight line $y = \dfrac{2}{3}x + 4$ to $2x - 3y - 8 = 0$.

    求直線 $y = \dfrac{2}{3}x + 4$ 與 $2x - 3y - 8 = 0$ 的最短距離。

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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Combinatorics
組合數學

21. Find the number of the combination(s) arranging 4 girls and 2 boys in a circle.
    求把 4 位女孩和 2 位男孩排成一圓的排列數目。

22. There are 5 identical Mathematics books, 4 identical Chinese books and 2 identical English books, how many different arrangement(s) is/are there?
    書架上有 5 本相同的數學書，4 本相同的中文書和 2 本相同的英文書，問有多少種排列方法？

23. A fair 6-face die is thrown 2 times. Find the probability that the sum of numbers obtained is a multiple of 3.
    擲一枚均質六面骰子兩次。求擲得點數總和為 3 的倍數的概率。

24. When does the hour hand and minute hand form a straight line in between 2 o'clock and 3 o'clock?
    在 2 時與 3 時之間，時針和分針什麼時間會成一直線？

25. Given (a,b,c) is a set of integers and all of them are greater than 3. Find the number of solution set(s) of $a+b+c=19$.
    已知 (a,b,c) 為一組整數且全部大於 3，求方程 $a+b+c=19$ 的解組數目。

~ 全卷完 ~
~ End of Paper ~