



奧冠教育中心

OLYMPIAD CHAMPION EDUCATION CENTRE

Room 309-310, 8 Jordan Road, Yau Ma Tei, Kowloon, Hong Kong SAR, CHINA

Tel (852) 3153 2028 / 9310 1240 Fax (852) 3153 2074

Website: www.olympiadchampion.com Email: olympiadchampion@gmail.com



香港國際數學競賽初賽 2019 (香港賽區)

HONG KONG INTERNATIONAL
MATHEMATICAL OLYMPIAD
HEAT ROUND 2019 (HONG KONG REGION)

中學二年級 Secondary 2

時限：90 分鐘

Time allowed: 90 minutes

模擬試題

Mock Paper

考生須知：

Instructions to Contestants:

1. 本卷包括 試題 乙份，試題紙不可取走。
Each contestant should have ONE Question-Answer Book which CANNOT be taken away.
2. 本卷共 5 個範疇，每範疇有 5 題，共 25 題，每題 4 分，總分 100 分，答錯不扣分。
There are 5 exam areas and 5 questions in each exam area. There are a total of 25 questions in this Question-Answer Book. Each carries 4 marks. Total score is 100 marks. No points are deducted for incorrect answers.
3. 請將答案寫在 答題紙 上。
All answers should be written on ANSWER SHEET.
4. 比賽期間，不得使用計算工具。
NO calculators can be used during the contest.
5. 本卷中所有圖形不一定依比例繪成。
All figures in the paper are not necessarily drawn to scale.
6. 比賽完畢時，本試題會被收回。
This Question-Answer Book will be collected at the end of the contest.

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

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All answers should be written on the ANSWER SHEET.

本試題不可取走。

THIS Question-Answer Book CANNOT BE TAKEN AWAY.

未得監考官同意，切勿翻閱試題，否則參賽者將有可能被取消資格。

DO NOT turn over this Question-Answer Book without approval of the examiner.

Otherwise, contestant may be DISQUALIFIED.

填空題 (第 1 至 25 題) (每題 4 分，答錯及空題不扣分)

Open-Ended Questions (1st ~25th) (4 points for correct answer, no penalty point for wrong answer)

Logical Thinking

邏輯思維

1. Given B and C are two non-zero digits and the 3-digit numbers formed by these two digits have the following properties:

1. \overline{BCB} is divisible by 7;
2. \overline{CBB} is divisible by 9;
3. \overline{BBC} has an odd number of factors.

Find the 3-digit number \overline{CCB} .

已知 A 和 C 為兩個非零數位，且利用這兩個數位組成的三位數有以下性質：

1. \overline{BCB} 可以被 7 整除；
2. \overline{CBB} 可以被 9 整除；
3. \overline{BBC} 的因數數目為單數。

求三位數 \overline{CCB} 。

2. Andy goes west for 31km, then goes north for 18km and goes east for 7km. How far is he now from the original position?

安迪向西走 31 公里，向北走了 18 公里，向東走了 7 公里，問他和原來位置相距多遠？

3. If $\overline{abcd} + \overline{dcba} = 16225$, calculate $a + b + c + d$.

若 $\overline{abcd} + \overline{dcba} = 16225$ ，求 $a + b + c + d$ 的值。

4. There are 28 problems in a mathematics competition. The scores of each problem are allocated in the following ways: 4 marks will be given for a correct answer, 0 marks will be given for a blank answer or wrong answer. Find the minimum number of candidate(s) to ensure that 3 candidates will have the same scores in the competition.

某數學比賽共有 28 條題目。以下述方式為每個題目評分：答對得 4 分、答錯或不作答得零分。求最小參賽者的數目才可保證比賽中有三人同分。

5. There are 8 pieces of white chopsticks, 10 pieces of yellow chopsticks and 7 pieces of brown chopsticks mixed together. Close your eyes. If you want to get a pair of chopsticks that is not white, at least how many piece(s) of chopstick(s) is / are needed to be taken?

現有白色、黃色、棕色的筷子分別有 8 根、10 根和 7 根，把它們混在一起。你要在閉上眼睛的情

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

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況下，從這些筷子中取出一雙不是白色的筷子，最少要取多少根？

Algebra

代數

6. Find the value of y if $|x + y + 9| + |x - y - 5| = 0$.

若 $|x + y + 9| + |x - y - 5| = 0$ ，求 y 的值。

7. Factorize $x^2 - y^2 - 4y + 2x - 3$.

因式分解 $x^2 - y^2 - 4y + 2x - 3$

8. How many integral solution(s) is / are there for x if $-33 \leq 5x + 3 \leq 26$?

若 $-33 \leq 5x + 3 \leq 26$ ，則 x 有多少個整數解？

9. Find the value of $(1 - \frac{1}{4})(1 - \frac{1}{9})(1 - \frac{1}{16}) \dots (1 - \frac{1}{4072324})$.

求 $(1 - \frac{1}{4})(1 - \frac{1}{9})(1 - \frac{1}{16}) \dots (1 - \frac{1}{4072324})$ 的值。

10. Given that a is a real number and $a \neq -1$. If $a + 2 + \frac{1}{a+1} = -1$, find the value of a .

已知 a 為實數且 $a \neq -1$ ，若 $a + 2 + \frac{1}{a+1} = -1$ ，求 a 的值。

Number Theory

數論

11. Find the remainder when 2018^{8012} is divided by 7.

求當 2018^{8012} 除以 7 時所得的餘數。

12. x is a positive integer such that the remainder of x^2 divided by 9 is 7. If $x > 23$, what is the minimal value of x ?

x 是一個正整數使 x^2 除以 9 餘 7。若 $x > 23$ ， x 最小是多少？

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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13. Find the last digit of A if $A = 1 + 4 + 4^2 + 4^3 + \dots + 4^{2017} + 4^{2018}$
若 $A = 1 + 4 + 4^2 + 4^3 + \dots + 4^{2017} + 4^{2018}$ ，求 A 的最後一個數位的值。

14. Find the sum of all positive factors of 2040.
求 2040 的正因數之和。

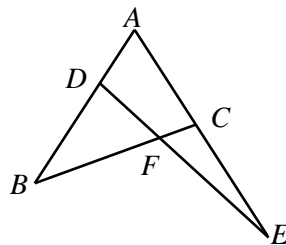
15. If $(x-2)$ is a factor of polynomial $(4x^2 - 77x + c)$. Find the value of c .
若 $(x-2)$ 是多項式 $(4x^2 - 77x + c)$ 的因式，求 c 的值。

Geometry

幾何

16. In the figure below, D is a point on AB . E is on the extension of AC . DE intersects BC at F . If $\frac{DE}{EF} = \frac{9}{4}$,
 $\frac{AD}{DB} = \frac{2}{3}$. Find the value of $\frac{AC}{AE}$.

在下圖中， D 是 AB 上的一點。延長 AC 至 E 點。 DE 與 BC 相交於 F 點。若 $\frac{DE}{EF} = \frac{9}{4}$ 及 $\frac{AD}{DB} = \frac{2}{3}$ ，
求 $\frac{AC}{AE}$ 的值。



Question 16

第 16 題

17. For three points on a coordinate plane $A(7, 8)$, $B(2, 5)$ and $C(3, 6)$, find the area of the triangle formed by using those three points as vertices.

直角坐標上有三個點分別為 $A(7, 8)$ 、 $B(2, 5)$ 及 $C(3, 6)$ ，求以該三點為頂點的三角形的面積。

18. An iron wire is bent into a circle which radius is 21. If the wire is now bent into a rectangle, what is the maximum value of area of it? (Take $\pi = \frac{22}{7}$)

一條鐵線被屈成一個半徑為 21 的圓形，若這條鐵線現在被屈成一個矩形，面積最大是多少？（取 $\pi = \frac{22}{7}$ ）

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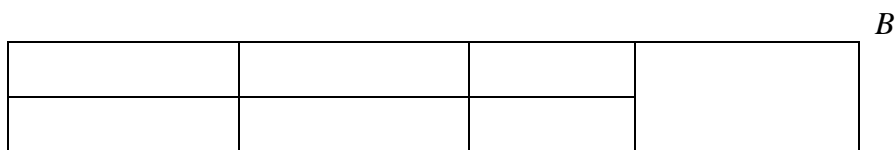
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19. An interior angle of a n -sided polygon is $(12n + 6)^\circ$. Find n .
一個正 n 邊形的其中一隻內角為 $(12n + 6)^\circ$ ，求 n 。
20. How many diagonal(s) does a convex decagon(10-sided polygon) have?
一個凸十邊形有多少條對角線？

Combinatorics

組合數學

21. Find the number of the combination(s) arranging 2 boys and 6 girls in a row.
求把 2 位男孩與 6 位女孩排成一列的排列數目。
22. 3 identical Mathematics books and 5 identical English books are put on the bookshelf. How many way(s) of arrangement is/are there?
3 本相同的數學書和 5 本相同的英文書放在書架上，共有多少個不同的排列方法？
23. A fair 6-face die is thrown 2 times. Find the probability that the product of numbers obtained is 12.
擲一枚均質六面骰子兩次。求擲得點數之積為 12 的概率。
24. Amy draws all triangles which perimeter is 26cm and length of sides are integers. How many different types of triangle(s) does she draw? (triangles with sides (4,5,6) and (4,6,5) are regarded as the same type of triangle)
艾美畫了所有周界為 26 厘米，且邊長為整數的三角形。問她共畫了多少種不同的三角形？(邊長 (4,5,6) 和 (4,6,5) 被視為同 1 種三角形)
25. If Anna goes from point A to point B, each step can only move up or move right. How many method(s) is / are there?
如果安娜從 A 點走到 B 點，每步只能向上或向右走，共有多少種走法？

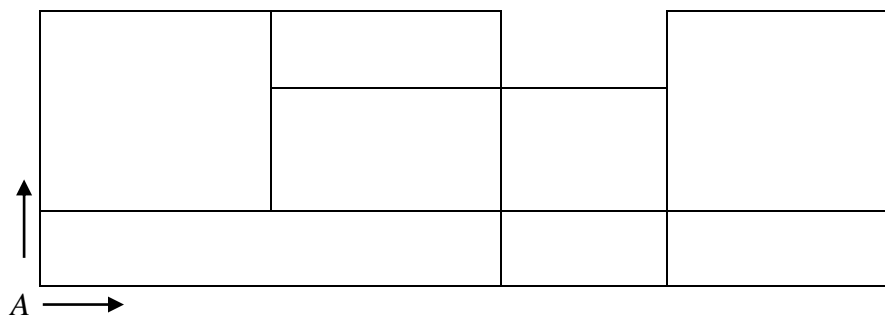


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Question 25

第 25 題

~ 全卷完 ~

~ End of Paper ~

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