香港國際數學競賽初賽 **2019**（香港賽區）

HONG KONG INTERNATIONAL MATHEMATICAL OLYMPIAD HEAT ROUND 2019 (HONG KONG REGION)

小學六年級 Primary 6

時限：90 分鐘

Time allowed: 90 minutes

模擬試題

Mock Paper

考生須知：

Instructions to Contestants:

1. 本卷包括 試題 乙份，試題紙不可取走。

   Each contestant should have ONE Question-Answer Book which CANNOT be taken away.

2. 本卷共 5 個範疇，每範疇有 5 題，共 25 題，每題 4 分，總分 100 分，答錯不扣分。

   There are 5 exam areas and 5 questions in each exam area. There are a total of 25 questions in this Question-Answer Book. Each carries 4 marks. Total score is 100 marks. No points are deducted for incorrect answers.

3. 請將答案寫在 答題紙 上。

   All answers should be written on ANSWER SHEET.

4. 比賽期間，不得使用計算工具。

   NO calculators can be used during the contest.

5. 本卷中所有圖形不一定依比例繪成。

   All figures in the paper are not necessarily drawn to scale.

6. 比賽完畢時，本試題會被收回。

   This Question-Answer Book will be collected at the end of the contest.

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

填空題（第 1 至 25 題）（每題 4 分，答錯及空題不扣分）

Open-Ended Questions (1st ~25th) (4 points for correct answer, no penalty point for wrong answer)

Logical Thinking
邏輯思維

1. Given A and D are two non-zero digits and the 2-digit numbers formed by these two digits have the following properties:
    1. $\overline{DA}$ can be expressed by a product of 2 and another prime numbers;
    2. $\overline{AD}$ can be expressed by a product of 2 and another prime numbers;
    If $A > D$, find the 2-digit number $\overline{AD}$.

    已知 A 和 D 為兩個非零數位。且利用這兩個數位組成的兩位數有以下性質：
    1. $\overline{DA}$ 可以被寫成 2 和一個質數的積；
    2. $\overline{AD}$ 可以被寫成 2 和一個質數的積；

    若 $A > D$，求兩位數 $\overline{AD}$。

2. There are chickens and rabbits in a cage and the number of rabbits is twice the number of chickens. They have 340 legs in total. How many rabbits are there?

    雞兔同籠，兔的數量是雞的 2 倍，共有腿 340 條，問兔有多少隻？

3. Amy is 10 years younger than his brother. Uncle's age is three times of the sum of Amy's and brother's age now. Six years later, Uncle's age is two times of the sum of Amy's and brother's age. Find the age of Amy now.

    艾美比哥哥少 10 歲，而伯伯的年齡是艾美和哥哥年齡之和的 3 倍。六年後，伯伯的年齡是艾美和哥哥年齡之和的 2 倍。求艾美現在的年齡。

4. John wrote a 4-digit number on a piece of paper and asked Peter to guess it.
    Peter asked: "Is the number 2956?"
    John replied: "Three of the digits are correct. The positions of those digits are all wrong."
    Peter asked again: "Is the number 7324?"
    John replied: "Two of the digits are correct. The positions of those digits are all wrong."
    Peter asked again: "Is the number 4962?"
    John said: "All the digits are correct. The positions of those digits are all wrong."
    Given all the digits in that 4-digit number are all different, what is the number written by John?

    約翰寫一個 4 位數在一張紙上，吩咐彼得來猜猜它。

    彼得問：「這個數是否 2956？」

約翰回覆：「這裏有三個數位是正確的，但是這些數位的位置都是錯誤的。」

彼得再問：「這個數是否 7324？」

約翰回覆：「這裏有兩個數位是正確的，但是這些數位的位置都是錯誤的。」

彼得再問：「這個數是否 4962？」

約翰回覆：「這裏所有數位都是正確的，但是這些數位的位置都是錯誤的。」

已知這個 4 位數的 4 個數位都不一樣，求約翰寫出來的數字。

5. There are 7 pieces of white chopsticks, 6 pieces of yellow chopsticks and 5 pieces of brown chopsticks mixed together. If you want to get 3 pairs of chopstick in the dark, in which 2 pairs of them are of the same color, at least how many piece(s) of chopstick(s) is / are needed to be taken?

現有白色、黃色、棕色的筷子分別有 7 根、6 根和 5 根，把它們混在一起。在黑暗中，你要從這些筷子中取出兩對相同顏色和一對不同顏色的筷子，最少要取多少根？

Arithmetic
算術

6. Find the value of $2222 \times 3333 + 3334 \times 1111$.

求 $2222 \times 3333 + 3334 \times 1111$ 的值。

7. Find the value of $\dfrac{1}{2 \times 4} + \dfrac{1}{4 \times 6} + ... + \dfrac{1}{2014 \times 2016} + \dfrac{1}{2016 \times 2018}$.

求 $\dfrac{1}{2 \times 4} + \dfrac{1}{4 \times 6} + ... + \dfrac{1}{2014 \times 2016} + \dfrac{1}{2016 \times 2018}$ 的值。

8. Find the value of $66 \times 65 - 65 \times 64 + 64 \times 63 - 63 \times 62 + ... + 4 \times 3 - 3 \times 2 + 2 \times 1$.

求 $66 \times 65 - 65 \times 64 + 64 \times 63 - 63 \times 62 + ... + 4 \times 3 - 3 \times 2 + 2 \times 1$ 的值。

9. Find the value of $199 + 198 + ... + 101 + 100 - 99 - 98 - 97 - ... - 1$.

求 $199 + 198 + ... + 101 + 100 - 99 - 98 - 97 - ... - 1$ 的值。

10. Convert $\dfrac{235}{999}$ into the decimals.

把 $\dfrac{235}{999}$ 轉化為小數。

Number Theory
數論

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。
Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

11. If 9-digit number $\overline{A2018901B}$ is divisible by 72, find the value of $A+B$.
    若 9 位數 $\overline{A2018901B}$ 可被 72 整除，求 $A+B$ 的值。

12. Find the last digit of A if $A = 8^2 + 18^2 + 28^2 + ... + 2018^2$.
    若 $A = 8^2 + 18^2 + 28^2 + ... + 2018^2$，求 A 的最後一個數位的值。

13. How many 3 digit number(s) that can be divisible by 2 and 3 but not divisible by 7?
    能被 2 和 3 整除而不能被 7 整除的三位數有多少個？

14. A 4-digit number has remainder 1 when divided by 3, has remainder 2 when divided by 5 and has remainder 3 when divided by 7. What is such largest 4 digit number?
    一個四位數除以 3 餘 1，除以 5 餘 2，除以 7 餘 3，問這個四位數最大是多少？

15. The sum of 8 consecutive odd numbers is 144. Find the largest odd number.
    某 8 個連續奇數之和是 144，求最大的奇數。

Geometry
幾何

16. How many rectangle(s) is / are there in the figure below?
   請問下圖有多少個長方形？
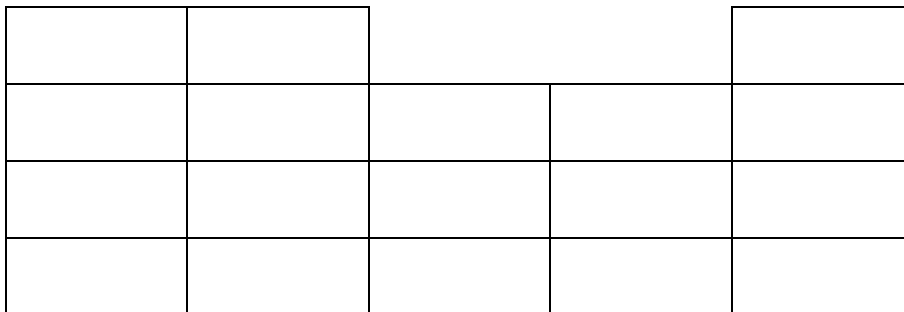
Question 16
第 16 題

17. A rectangle is cut into 4 equal rectangles. If the area of it is 252cm^2, how many cm at most does the perimeter increase in cm after cutting the rectangle?
    一個長方形被切割成 4 份。若長方形的面積為 252 平方厘米，切割之後周界最多增加了多少厘米？

18. Small cubes with side length 1cm are combined according to the pattern shown below, if there are 12 layers, find the volume.
    邊長為 1 的小立方體按以下規律組合成立體，如果它有 12 層，求該立體的體積。
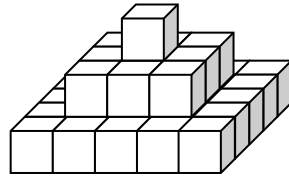
Question 18
第 18 題

19. A big rectangle is formed by 1001 squares with side length 2cm. Find the minimum value of the perimeter in cm.

1001 個邊長 2 厘米的正方形砌成一個長方形，求長方形周界的最小值。

20. The length of side of a square is 12.56cm. If the perimeter of the square is the same as a circle, what is the radius of the circle? (Take $\pi = 3.14$)

一個正方形的邊長為 12.56 厘米。若正方形的周界和一個圓形相同，求該圓形的半徑。(取 $\pi = 3.14$)

Combinatorics
組合數學

21. A flight of stairs has 10 steps. David can go up for 1 step or 2 steps each time. The 5th step cannot be stepped on as it is destroyed. How many way(s) is / are there for David to go up the stairs?

一道有 10 級的樓梯，大衛每一步可以上 1 級或 2 級。其中第 5 級因為損壞而不能踏上，問大衛走上這道樓梯共有多少種方法？

22. Amy needs 12 days to finish a task. Paul needs 24 days to finish a task. How many days do they need to finish the task if they do it together?

艾美需要 12 日完成一份工作，保羅需要 24 天完成同一份工作。若他們合作完成工作，需要多少天？

23. When is the hour hand and the minute hand overlap between 5pm and 6pm?

在下午 5 時至 6 時之間，時針和分針何時重疊？

24. Numbers are drawn from 2018 integers 1 to 2018. At least how many number(s) is / are drawn at random to ensure that there are two numbers which difference is 2901?

在 1 至 2018 這 2018 整數中最少任意選出多少個，才必定有兩個數之差是 901？

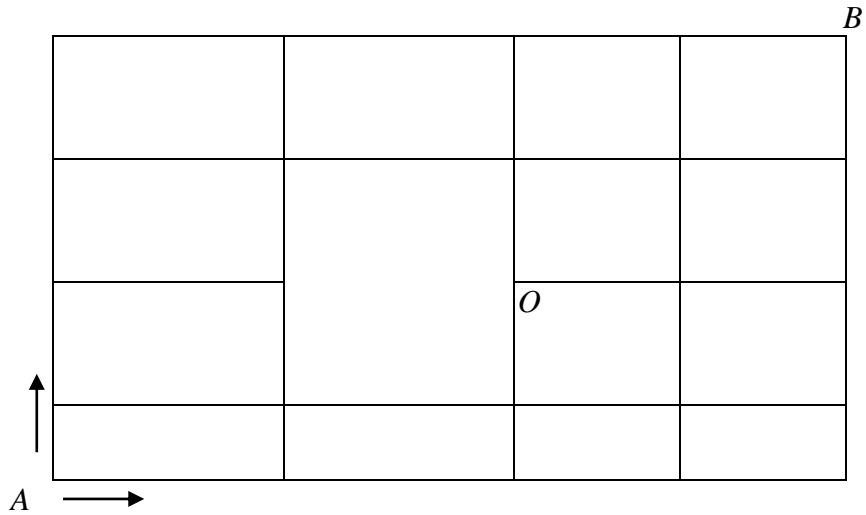請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

25. If Agnes goes from point *A* to point *B*, each step can only move up or move right. How many way(s) is / are there if she must pass through the point *O*?

如果雅麗從 *A* 點走到 *B* 點，每步只能向上或向右走，且必須經過 *O* 點，共有多少種走法？

B

O

A →

Question 25

第 25 題

~ 全卷完 ~

~ End of Paper ~