香港國際數學競賽初賽 **2019**（香港賽區）

HONG KONG INTERNATIONAL MATHEMATICAL OLYMPIAD HEAT ROUND 2019 (HONG KONG REGION)

小學五年級 Primary 5

時限：90 分鐘

Time allowed: 90 minutes

模擬試題
Mock Paper

考生須知：

Instructions to Contestants:

1. 本卷包括 試題 乙份，試題紙不可取走。
    Each contestant should have ONE Question-Answer Book which CANNOT be taken away.

2. 本卷共 5 個範疇，每範疇有 5 題，共 25 題，每題 4 分，總分 100 分，答錯不扣分。
    There are 5 exam areas and 5 questions in each exam area. There are a total of 25 questions in this Question-Answer Book. Each carries 4 marks. Total score is 100 marks. No points are deducted for incorrect answers.

3. 請將答案寫在 答題紙 上。
    All answers should be written on ANSWER SHEET.

4. 比賽期間，不得使用計算工具。
    NO calculators can be used during the contest.

5. 本卷中所有圖形不一定依比例繪成。
    All figures in the paper are not necessarily drawn to scale.

6. 比賽完畢時，本試題會被收回。
    This Question-Answer Book will be collected at the end of the contest.

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

本試題不可取走。

THIS Question-Answer Book CANNOT BE TAKEN AWAY.

未得監考官同意，切勿翻閱試題，否則參賽者將有可能被取消資格。

DO NOT turn over this Question-Answer Book without approval of the examiner.
Otherwise, contestant may be DISQUALIFIED.

填空題（第 1 至 25 題）（每題 4 分，答錯及空題不扣分）

Open-Ended Questions (1st ~25th) (4 points for correct answer, no penalty point for wrong answer)

<u>Logical Thinking</u>
<u>邏輯思維</u>

1. Given *A* and *B* are two non-zero digits and the 2-digit numbers formed by these two digits have the following properties:
    1. $\overline{AB}$ is divisible by 17;
    2. $\overline{BA}$ is a product of 2 odd prime numbers;
    Find the 2-digit number $\overline{BA}$.

    已知 *A* 和 *B* 為兩個非零數位，且利用這兩個數位組成的兩位數有以下性質：
    1. $\overline{AB}$ 可以被 17 整除；
    2. $\overline{BA}$ 是 2 個奇質數之積；

    求兩位數 $\overline{BA}$。

2. There are some chickens and rabbits, where the number of chickens is 28 more than the number of rabbits. If there are 338 legs in total, how many chickens are there?

    雞兔同籠，雞比兔多 28 隻，共有腿 338 條。問雞有多少隻？

3. A stationery store had a batch of ball pens. One half and 5 more pens were sold on the first day. One half of the remaining part but 12 fewer pens were sold on the second day. Finally 68 ball pens are left. How many ball pen(s) did the stationery store have originally?

    文具店有一批原子筆，第一天賣掉了總數的一半多 5 支，第二天則賣掉剩下的一半少 12 支，最後剩下 68 支。那麼文具店原有多少支原子筆？

4. John wrote a 4-digit number on a piece of paper and asked Peter to guess it.
    Peter asked: "Is the number 2435?"
    John replied: "Two of the digits are correct. The positions of those digits are also correct."
    Peter asked again: "Is the number 6397?"
    John replied: "Two of the digits are correct. The positions of those digits are all wrong."
    Peter asked again: "Is the number 9823?"
    John said: "All the digits are correct. The positions of those digits are all wrong."
    Given all the digits in that 4-digit number are all different, what is the number written by John?

    約翰寫一個 4 位數在一張紙上，吩咐<u>彼得</u>來猜猜它。

    <u>彼得</u>問：「這個數是否 2435？」

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

約翰回覆：「這裏有兩個數位是正確的，而且這些數位的位置也是正確的。」

彼得再問：「這個數是否 6397？」

約翰回覆：「這裏有兩個數位是正確的，但是這些數位的位置都是錯誤的。」

彼得再問：「這個數是否 9823？」

約翰回覆：「這裏所有數位都是正確的，但是這些數位的位置都是錯誤的。」

已知這個 4 位數的 4 個數位都不一樣，求約翰寫出來的數字。

5. There are 5 blue balls, 7 red balls and 6 green balls in a bag. At least how many balls should be picked up to ensure there are 3 balls for each color?

一個袋內有 5 個藍球，7 個紅球和 6 個綠球。最少要抽多少個球才能確保取得每種顏色的球各 3 個？

<div align="center">

Arithmetic

算術

</div>

6. Find the value of $1001+143\times24+429\times3$.

求 $1001+143\times24+429\times3$ 的值。

7. Find the value of $\dfrac{1}{7\times8}+\dfrac{1}{8\times9}+\dfrac{1}{9\times10}+...+\dfrac{1}{26\times27}+\dfrac{1}{27\times28}$.

求 $\dfrac{1}{7\times8}+\dfrac{1}{8\times9}+\dfrac{1}{9\times10}+...+\dfrac{1}{26\times27}+\dfrac{1}{27\times28}$ 的值。

8. Find the value of $2018+2017-2016+2015+2014-2013+...+5+4-3+2+1$.

求 $2018+2017-2016+2015+2014-2013+...+5+4-3+2+1$ 的值。

9. Find the value of 667^2-333^2.

求 667^2-333^2 的值。

10. Find the value of $\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\dfrac{1}{16}+\dfrac{1}{32}+\dfrac{1}{64}+\dfrac{1}{128}+\dfrac{1}{256}$.

求 $\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\dfrac{1}{16}+\dfrac{1}{32}+\dfrac{1}{64}+\dfrac{1}{128}+\dfrac{1}{256}$ 的值。

<div align="center">

Number Theory

數論

</div>

11. If 9-digit number $\overline{2018A902B}$ is divisible by 144, find the value of $A+B$.

若 9 位數 $\overline{2018A902B}$ 可被 144 整除，求 $A+B$ 的值。

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

12. Find the number of factors for 720.
    問 720 有多少個因數？

13. Define the operation symbol $a \otimes b = a \times b - a - b + 1$, find the value of $13 \otimes (9 \otimes 6)$.
    定義運算 $a \otimes b = a \times b - a - b + 1$，求 $13 \otimes (9 \otimes 6)$ 的值。

14. The sum of two positive numbers *A* and *B* is 96. Find the maximum value of the product of *A* and *B*.
    老師將若干粒糖果分給學生，若每人派 4 粒糖果，會剩下 87 粒；若每人派 9 粒糖果，會剩下 12 粒。問老師共有多少粒糖？

15. How many 3-digit number(s) that can be divisible by 4 but cannot be divisible by 7 is / are there?
    有多少個三位數能被 4 整除而不能被 7 整除。

Geometry
幾何

16. How many rectangle(s) is / are there in the figure below?
    請問下圖有多少個長方形？
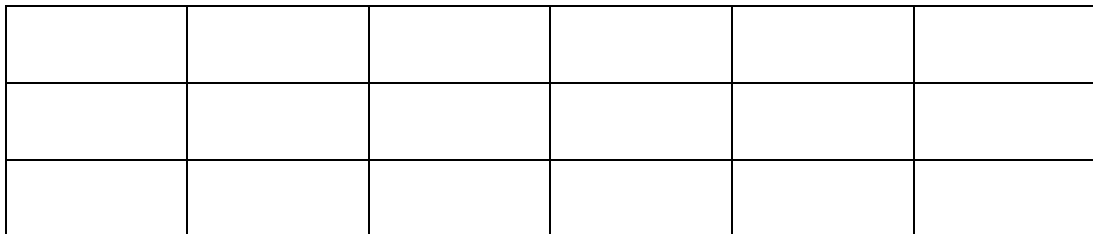
Question 16
第 16 題

17. As shown in the figure below, area of ΔABC is 960 cm². If *D*, *E* and *F* are midpoints of *AC*, *BC* and *BE* respectively, find the area of the shaded region in cm².
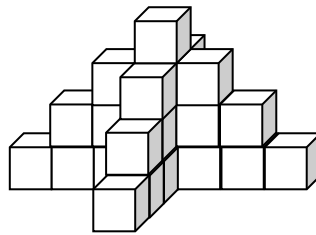   圖中 ΔABC 的面積是 960 平方厘米，*D* 和 *E* 分別是 *AC* 和 *BC* 的中點，*F* 是 *BE* 中點，問陰影部份的面積是多少平方厘米？
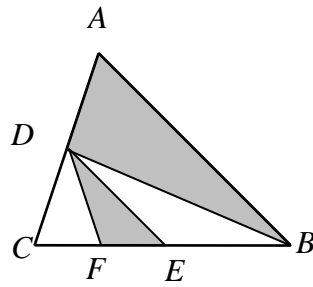
Question 17

第 17 題

18. 28 small cubes with side length 1cm are combined according to the pattern shown below, find the surface area in cm^2.

28 個邊長為 1 厘米的小立方體按以下規律組合成立體，求表面面積（以平方厘米表示答案）。
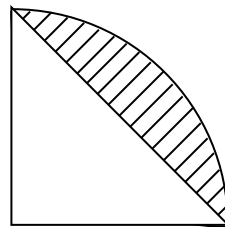

Question 18

第 18 題

19. How many diagonals does a heptagon(7-sided polygon) have?

七邊形共有多少條對角線？

20. Find the area of the shaded region in the figure. (Take $\pi = \dfrac{22}{7}$)

下圖中陰影部份面積是多少？（取 $\pi = \dfrac{22}{7}$）

42

Question 20

第 20 題

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

Combinatorics
組合數學

21. Amy draws all triangles which perimeter is 16cm and length of sides are integers. How many different types of triangle(s) does she draw? (triangles with sides (5,5,6) and (5,5,6) are regarded as the same type of triangle)
艾美畫了所有周界為 16 厘米,且邊長為整數的三角形。問她共畫了多少種不同的三角形?(邊長(5,5,6)和(5,6,5)被視為同 1 種三角形)

22. How many digits are there from page numbers from page 23 to page 1028 of a book (inclusively)?
一本書由第 23 頁到第 1028 頁之間的頁碼有多少個數位(包括第 23 頁和第 1028 頁)?

23. At least how many of 253 children that was/were born in the same Chinese zodiac year? (There are 12

請以最簡形式填寫答案,若計算結果是分數,請確保為真分數或帶分數,或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

different Chinese zodiac year)

253 個人之中最少有多少個是在同一個生肖年出生？

24. Numbers are drawn from 123 integers 1 to 123. At least how many number(s) is / are drawn at random to ensure that there are two numbers whose difference is 70?

在 1 至 123 這 123 個整數中最少任意選出多少個數，才必定有兩個數之和是 70？

25. If Alice goes from point *A* to point *B*, each step can only move up or move right. How many way(s) is / are there?

如果愛麗絲從 *A* 點走到 *B* 點，每步只能向上或向右走，共有多少種走法？
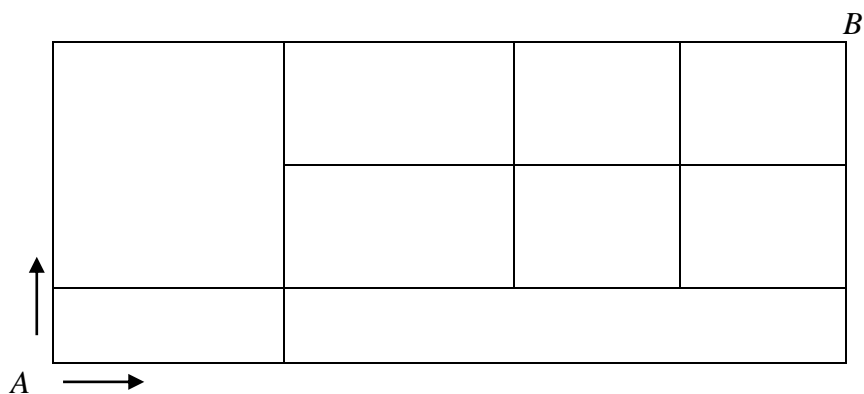
Question 25

第 25 題

~ 全卷完 ~

~ End of Paper ~

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.