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香港國際數學競賽初賽 2019（香港賽區）

HONG KONG INTERNATIONAL MATHEMATICAL OLYMPIAD HEAT ROUND 2019 (HONG KONG REGION)

高中組 Senior Secondary

時限：90 分鐘

Time allowed: 90 minutes

模擬試題

Mock Paper

考生須知：

Instructions to Contestants:

1. 本卷包括 試題 乙份，試題紙不可取走。
   Each contestant should have ONE Question-Answer Book which CANNOT be taken away.
2. 本卷共 5 個範疇，每範疇有 5 題，共 25 題，每題 4 分，總分 100 分，答錯不扣分。
   There are 5 exam areas and 5 questions in each exam area. There are a total of 25 questions in this Question-Answer Book. Each carries 4 marks. Total score is 100 marks. No points are deducted for incorrect answers.
3. 請將答案寫在 答題紙 上。
   All answers should be written on ANSWER SHEET.
4. 比賽期間，不得使用計算工具。
   NO calculators can be used during the contest.
5. 本卷中所有圖形不一定依比例繪成。
   All figures in the paper are not necessarily drawn to scale.
6. 比賽完畢時，本試題會被收回。
   This Question-Answer Book will be collected at the end of the contest.

請將答案寫在 答題紙 上。
All answers should be written on the ANSWER SHEET.

本試題不可取走。
THIS Question-Answer Book CANNOT BE TAKEN AWAY.
未得監考官同意，切勿翻閱試題，否則參賽者將有可能被取消資格。
DO NOT turn over this Question-Answer Book without approval of the examiner.
Otherwise, contestant may be DISQUALIFIED.

填空題（第 1 至 25 題）(每題 4 分，答錯及空題不扣分）

Open-Ended Questions (1^{st} ~25^{th}) (4 points for correct answer, no penalty point for wrong answer)

Logical Thinking
邏輯思維

1. Given A and B are three non-zero digits and the 3-digit numbers formed by these three digits have the following properties:
   1. $\overline{BBA}$ has an odd number of factors;
   2. $\overline{ABB}$ has an odd number of factors.

   Find the 3-digit number $\overline{ABA}$.

   已知 A 和 B 為三個非零數位且利用這三個數位組成的三位數有以下性質：

   1. $\overline{BBA}$ 的因數數目為單數；
   2. $\overline{ABB}$ 的因數數目為單數。

   求三位數 $\overline{ABA}$。

2. Given that the mean, median, range and the only mode of 300 integers are also 168. If A is the smallest integer among those 300 integers, find the minimum value of A.

   已知 300 個整數的平均數、中位數、分佈域及唯一眾數皆為 168。若 A 為該 300 個整數中的最小整數，求 A 的最小值。

3. Andy goes northwest for 24km, then goes northeast for 13km, goes southeast for 51km and goes southwest for 49km. How far is he now from the original position?

   安迪向西北走了 24 公里，向東北走了 13 公里，向東南走了 51 公里，向西南走了 49 公里，問他和原來位置相距多遠？

4. There are 20 problems in a mathematics competition. The scores of each problem are allocated in the following ways: 3 marks will be given for a correct answer. 1 mark will be deducted from a wrong answer and 0 marks will be given for a blank answer. Find the minimum number of candidate(s) to ensure that 2 candidates will have the same scores in the competition.

   某數學比賽共有 20 條題目。以下述方式為每個題目評分：答對得 3 分、答錯倒扣 1 分、不作答得零分。求最小參賽者的數目才可保證比賽中有兩人同分。

5. There are 13 pieces of white chopsticks, 18 pieces of yellow chopsticks and 23 pieces of brown chopsticks mixed together. Close your eyes. If you want to get 2 pairs of chopsticks that are not brown, at

least how many piece(s) of chopstick(s) is / are needed to be taken?
現有白色、黃色、棕色的筷子分別有 13 根、18 根和 23 根，把它們混在一起。你要在閉上眼睛的情況下，從這些筷子中取出 3 雙不是棕色的筷子，最少要取多少根？

Algebra
代數

6. Let α and β ($\alpha,\beta \neq 0$) be the roots of the equation $x^2 - 2018x + 1 = 0$. If $\frac{\beta}{\alpha}$ and $\frac{\alpha}{\beta}$ are the roots of the equation $x^2 - Sx + 1 = 0$, find the value of S.
已知$\alpha,\beta \neq 0$，設方程 $x^2 - 2018x + 1 = 0$ 的根為α 和 β。若$\frac{\beta}{\alpha}$和$\frac{\alpha}{\beta}$是方程$x^2 - Sx + 1 = 0$的根，求 S 的值。

7. Find the constant term in the expansion of $(x - \frac{2}{x})^4 (x + \frac{1}{x})^3$.
求$(x - \frac{2}{x})^4 (x + \frac{1}{x})^3$展開式中的常數項。

8. Calculate $\sqrt{3+\sqrt{3+\sqrt{3+...}}}$.
計算$\sqrt{3+\sqrt{3+\sqrt{3+...}}}$。

9. If a is a positive integer, find the smallest value of a such that $(a-2)x^2 + (a-1)x + a = 0$ has no real roots.
若 a 為正整數，求 a 的最小值使得$(a-2)x^2 + (a-1)x + a = 0$沒有實根。

10. $f(x)$ is a degree 2 polynomial function. If $f(0) = -4$, $f(1) = -1$, $f(2) = 4$, find $f(x)$.
$f(x)$是一個 2 次多項式函數，若 $f(0) = -4$ ， $f(1) = -1$ ， $f(2) = 4$ ，求 $f(x)$ 。

Number Theory
數論

11. Given that $\overline{20A18B}$ is a 6-digit number which is divisible by 132, find the value of $A + B$.

請將答案寫在 答題紙 上。
All answers should be written on the ANSWER SHEET.

已知 $\overline{20A18B}$ 是一個六位數，且可被 132 整除，求 $A+B$ 的值。

12. Now is May. Which month will it be after 8^{26} months?
現在是五月，8^{26} 個月後是幾月？

13. How many positive integer(s) x is / are there so that $\sqrt{x^2+4x+670}$ is an integer?
有多少個正整數 x 使 $\sqrt{x^2+4x+670}$ 為整數？

14. For a natural number, after subtracting 15, it becomes a square number. It also becomes a square number if it adds 69. Find such largest natural number.
有一個自然數，它減去 15 後是平方數，它加上 69 也是一個自然數，求這樣的最大自然數。

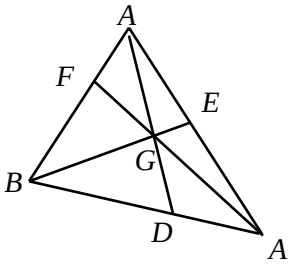
15. What is the largest integral value n that satisfies the inequality $n^{320} \le 8^{240}$?
求能滿足不等式 $n^{320} \le 8^{240}$ 的整數的最大值。

請將答案寫在 答題紙 上。
All answers should be written on the ANSWER SHEET.

Geometry
幾何

16. Find the area enclosed by the *y*-axis and the straight lines $3x+y=0$ and $2x+y-12=0$.
求由 *y*-軸及直線 $3x+y=0$ 及 $2x+y-12=0$ 圍出的面積。

17. In the figure below, *AD*, *BE* and *CF* intersect at a point *G*.. If $\frac{AF}{FB}=\frac{2}{3}$, $\frac{CD}{BC}=\frac{3}{7}$. Find the value of $\frac{AE}{AC}$
在下圖中，*AD*、*BE* 和 *CF* 相交於一點 *G*。若 $\frac{AF}{FB}=\frac{2}{3}$ 及 $\frac{CD}{BC}=\frac{3}{7}$，求 $\frac{AE}{AC}$ 的值。



Question 17
第 17 題

18. In ΔABC, $a=6$, $c=7$, $\cos C=\frac{29}{40}$. Find *b*.
ΔABC 中，$a=6$，$c=7$，$\cos C=\frac{29}{40}$，求 *b*。

19. Given that $\cos x-\sin x=\frac{1}{2}$. Find the value of $\sin x\cos x$.
已知 $\cos x-\sin x=\frac{1}{2}$，求 $\sin x\cos x$ 的值。

20. Find the shortest distance from the point $(9,5)$ to straight line $8x-6y+13=0$.
求點 $(9,5)$ 與直線 $8x-6y+13=0$ 的最短距離。

請將答案寫在 答題紙 上。
All answers should be written on the ANSWER SHEET.

Combinatorics
組合數學

21. Find the number of the combination(s) arranging 5 girls and 2 boys in a circle.
求把 5 位女孩和 2 位男孩排成一圓的排列數目。

22. Amy draws all triangles which perimeter is 31cm and length of sides are integers. How many different types of triangle(s) does she draw? (triangles with sides (4,5,6) and (4,6,5) are regarded as the same type of triangle)
艾美畫了所有周界為 31 厘米，且邊長為整數的三角形。問她共畫了多少種不同的三角形？（邊長（4,5,6）和（4,6,5）被視為同 1 種三角形）

23. A fair 6-face die is thrown 3 times. Find the probability that the sum of numbers obtained is a multiple of 5.
擲一枚均質六面骰子三次。求擲得點數總和為 5 的倍數的概率。

24. There are 7 identical Mathematics books, 2 identical English books and 4 identical Chinese books. How many different arrangement(s) is/are there?
在書架上有 7 本相同的數學書，2 本相同的英文書，4 本相同的中文書，共有多少種排列方法？

25. Given (a,b,c,d) is a set of integers and $a \geq 2, b \geq 1, c \geq 0, d \geq -1$. Find the number of solution set(s) of $a+b+c+d=0$.
已知 (a,b,c,d) 為一組整數且 $a \geq 2, b \geq 1, c \geq 0, d \geq -1$，求方程 $a+b+c+d=5$ 的解組數目。

~ 全卷完 ~
~ End of Paper ~